CLASS - XI

Chapter – 9

SEQUENCES AND SERIES

MODULE – 2 of 3

Distance Learning Programme: An initiative by AEES, Mumbai

5. Arithmetic Progression (A.P.)

Sequence of numbers such that the difference of any two successive members of the sequence is a constant

Examples of some A.P

- 1, 1, 1, 1, 1, 1, 1, ……….
- 5, 7, 9, 11, 13, 15, 17
- 100, 90, 80, 70, 60, 50
- -7, -3, 1, 5, 9, 13, 17, ……..
- $p, p + q, p + 2q, p + 3q, p + 4q$

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ is *arithmetic sequence* or *arithmetic progression.*
Here a_1 is the *first term (a)* and the constant difference between consecutive terms is called the *common difference (d)* of the A.P.

So, if first term is a and common difference is d then
A.P $\rightarrow a, a + d, a + 2d, a + 3d, a + 4d, \ldots$

Here in A.P.

1^{st} term $\rightarrow a$	*(in 1^{st} term 0d)*
2^{nd} term $\rightarrow a + d$	*(in 2^{nd} term 1d)*
3^{rd} term $\rightarrow a + 2d$	*(in 3^{rd} term 2d)*
4^{th} term $\rightarrow a + 3d$	*(in 4^{th} term 3d)*
5^{th} term $\rightarrow a + 4d$	*(in 5^{th} term 4d)*
6^{th} term $\rightarrow a + 5d$	*(in 6^{th} term 5d)*
7^{th} term $\rightarrow a + 6d$	*(in 7^{th} term 6d)*
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15^{th} term $\rightarrow a + 14d$	*(in 15^{th} term 14d)*
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n^{th} term $\rightarrow a + (n-1)d$	*(in n^{th} term (n-1)d)*

Hence,
The General term of an A.P is

$$a_n = a + (n - 1)d$$

Sum of terms in A.P.

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ is *arithmetic sequence* or *arithmetic progression.*

If S_n be the sum of first n terms of the A.P

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$

$S_n = a + (a + d) + (a + 2d) + \ldots.. a_n$

$S_n = a_n + (a_n - d) + (a_n - 2d) + \ldots.. + \quad a$

On adding equation (1) and (2)

$2S_n = (a + a_n) + (a + a_n) + (a + a_n) + \ldots.. + (a + a_n)$

$2S_n = (a + a_n) \times n$

$S_n = \frac{n}{2}\,(a + a_n)$

$S_n = \frac{n}{2}\,\{2a + (n-1)d\}$

Some properties of an A.P

(i) If a constant is added to each term of an A.P., the resulting sequence is also an A.P.

(ii) If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

(iii) If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

(iv) If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

Some more examples

Example 1

The sum of n terms of two arithmetic progressions are in the ratio $(3n + 8) : (7n + 15)$. Find the ratio of their 12th terms.

Solution

Let a_1, a_2, and d_1, d_2 be the first terms and common difference of the first and second arithmetic progression, respectively.

We have to find $\frac{12th\ term\ of\ 1s\ \ A.P}{12th\ term\ of\ 2nd\ A.P}$

Or

$$\frac{a_1 + \ 11d_1}{a_2 + \ 11d_2} = ?$$

According to the given condition, we have

$$\frac{Sum\ of\ first\ n\ terms\ of\ 1st\ A.P}{Sum\ of\ first\ n\ terms\ of\ 1st\ A.P} = \frac{3n+8}{7n+15}$$

$$\frac{\frac{n}{2}\{2a_1 + \ (n-1)d_1\}}{\frac{n}{2}\{2a_2 + \ (n-1)d_2\}} = \frac{3n+8}{7n+15}$$

To find the required ratio, put n = 23 ***(12^th^ term × 2 – 1)***

$$\frac{\frac{23}{2}\{2a_1 + (23-1)d_1\}}{\frac{23}{2}\{2a_2 + (23-1)d_2\}} = \frac{3\times 23+8}{7\times 23+15}$$

$$\frac{2a_1 + 22d_1}{2a_2 + 22d_2} = \frac{69+8}{161+15}$$

$$\frac{2(a_1 + 11d_1)}{2(a_2 + 11d_2)} = \frac{77}{176}$$

$$\frac{(a_1 + 11d_1)}{(a_2 + 11d_2)} = \frac{7}{16}$$

$$\frac{12th\ term\ of\ 1st\ A.P}{12th\ term\ of\ 2nd\ A.P} = \frac{7}{16}$$

Hence, the required ratio is 7 : 16.

6. Arithmetic mean (A.M.)

- **One Arithmetic Mean**

  Given two numbers a and b. We can insert a number A between them so that a, A, b is an A.P. Such a number A is called the *arithmetic mean* (A.M.) of the numbers a and b.

  As a, A, b is in A.P

So,

$$A - a = b - A$$

$$2A = a + b$$

$$A = \frac{a+b}{2}$$

Hence, Arithmetic mean of the numbers a and b is $\frac{a+b}{2}$

Example

Find the A.M. of two numbers 4 and 16.

Solution

$$A = \frac{a+b}{2} = \frac{4+16}{2} = \frac{20}{2} = 10$$

Arithmetic Mean is 10

Hence, 4, 10, 16 is in A.P

- ***n* Arithmetic Means**

  Given two numbers a and b. We can insert n numbers $A_1, A_2, A_3, \ldots\ldots, A_n$ between them so that $a, A_1, A_2, A_3, \ldots\ldots, A_n, b$ is an A.P.

  Here,
  1st term is a and $(n+2)th$ term is b.
  $a_{n+2} = a + (n + 2 - 1)d$
  $b = a + (n + 1)d$

$$d = \frac{b-a}{n+1}$$

Now

$$A_1 = a + d = a + \frac{b-a}{n+1} = \frac{na \quad b}{n+1}$$

$$A_2 = a + 2d = a + 2\,\frac{b-a}{n+1} = \frac{(n-1)a + 2b}{n+1}$$

$$A_3 = a + 3d = a + 3\,\frac{b-a}{n+1} = \frac{(n-2)a + 3b}{n+1}$$

$$A_4 = a + 4d = a + 4\,\frac{b-a}{n+1} = \frac{(n-3)a + 4b}{n+1}$$

$$A_n = a + nd = a + n\,\frac{b-a}{n+1} = \frac{a + nb}{n+1}$$

Hence, *n* arithmetic mean between *a* and *b* are

$$\frac{na \quad b}{n+1}, \frac{(n-1)a + 2b}{n+1}, \frac{(n-2)a + 3b}{n+1}, \frac{(n-3)a + 4b}{n+1}, \ldots\ldots\ldots, \frac{a + nb}{n+1}$$

Example

Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Solution

Let A_1, A_2, A_3, A_4, A_5 and A_6 be six numbers between 3 and 24.

Here, $a = 3$, $b = 24$ & $n = 6$ (6 Arithmetic means)

$$A_1 = \frac{na+b}{n+1} = \frac{\mathbf{6} \times 3 + 1 \times 24}{6+1} = \frac{18+24}{7} = \frac{42}{7} = 6$$

$$A_2 = \frac{\mathbf{5} \times 3 + 2 \times 24}{6+1} = \frac{15+48}{7} = \frac{63}{7} = 9$$

$$A_3 = \frac{\mathbf{4} \times 3 + 3 \times 24}{6+1} = \frac{12+7}{7} = \frac{84}{7} = 12$$

$$A_4 = \frac{\mathbf{3} \times 3 + 4 \times 24}{6+1} = \frac{9+96}{7} = \frac{105}{7} = 15$$

$$A_5 = \frac{\mathbf{2} \times 3 + 5 \times 24}{6+1} = \frac{6+120}{7} = \frac{126}{7} = 18$$

$$A_6 = \frac{\mathbf{1} \times 3 + 6 \times 24}{6+1} = \frac{3+144}{7} = \frac{147}{7} = 21$$

Hence, six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.